Class: XII

SESSION : 2022-2023

SUBJECT: Mathematics SAMPLE QUESTION PAPER - 19 with SOLUTION

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions :

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's and 02** Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)-type** questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)-type** questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)-type** questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

Section A

1. If a line with direction ratio 2 : 1 : 1 intersects the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z+2}{3}$ at A and B then $|\overrightarrow{AB}|$ is: [1]

   a) $2\sqrt{3}$ b) $2\sqrt{5}$

   c) $2\sqrt{7}$ d) $2\sqrt{6}$

2. $\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ dx = ? [1]

   a) $2x \tan^{-1}x - \log(1 + x^2) + C$ b) $-2x \tan^{-1}x - 2\log(1 + x^2) + C$

   c) None of these d) $2x \tan^{-1}x + \log(1 + x^2) + C$

3. If the vectors $\hat{i} - 2x\hat{j} + 3y\hat{k}$ and $\hat{i} + 2x\hat{j} - 3y\hat{k}$ are perpendicular to each other, then the locus of (x, y) is [1]

   a) an ellipse b) a circle

   c) none of these d) a hyperbola

4. India play two matches each with West Indies and Australia. In any match the probabilities of India getting 0,1 and 2 points are 0.45, 0.05 and 0.50 respectively. Assuming that the outcomes are independent, the probability of India getting at least 7 points is [1]

   a) 0.0875 b) 0.1125

   c) none of these d) $\frac{1}{16}$

5. If it is given that A and B are two events such that P(B) = $\frac{3}{5}$, $P(\frac{A}{B}) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$, $P(B|A')$ is equal to [1]

a) $\frac{1}{5}$

b) $\frac{3}{5}$

c) $\frac{1}{2}$

d) $\frac{3}{10}$

6. $\int \frac{dx}{(4x^2-4x+3)}$ =? [1]

a) None of these

b) $\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right)+C$

c) $-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right)+C$

d) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right)+C$

7. The area enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is equal to [1]

a) π^2ab

b) πab

c) πab^2

d) πa^{2}b

8. Forces 3 $\vec{OA}$, 5 $\vec{OB}$ act along OA and OB. If their resultant passes through C on AB, then [1]

a) 3 AC = 5 CB

b) divides AB in the ratio 2 : 1

c) 2 AC = 3CB

d) C is a mid-point of AB

9. The direction ratios of the line x - y + z - 5 = 0 = x - 3y - 6 are proportional to [1]

a) 3, 1, - 2

b) $\frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \frac{-2}{\sqrt{14}}$

c) 2, - 4, 1

d) $\frac{2}{\sqrt{41}}, \frac{-4}{\sqrt{41}}, \frac{1}{\sqrt{41}}$

10. The solution of the differential equation $2x \cdot \frac{dy}{dx} - y = 3$ represents a family of [1]

a) circles

b) parabolas

c) straight lines

d) ellipses

11. The area of the region bounded by the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$is [1]

a) $20\pi^2$ sq. units

b) 25π sq. units

c) 20π sq. units

d) $16\pi^2$ sq. units

12. $\int$ x tan^{-1} x dx = ? [1]

a) $\frac{1}{2}\tan^{-1}x + \log(1+x^2) - \frac{1}{2}x + C$

b) None of these

c) $\frac{1}{2}x^2\tan^{-1}x + \frac{1}{2}x + C$

d) $\frac{1}{2}(1+x^2)\tan^{-1}x - \frac{1}{2}x + C$

13. If $A = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 3 \\ -2 & -3 & 0 \end{bmatrix}$, $A + 2A^t$ equals [1]

a) $2A^2$

b) A^t

c) A

d) $-A^t$

14. Adj.(KA) = ______ [1]

a) K^{n-1} Adj. A

b) None of these

c) K Adj. A

d) K^n Adj.A

15. f(x) = sin x is increasing in [1]

a) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

b) $\left(\pi, \frac{3\pi}{2}\right)$

c) $(0, \pi)$

d) $\left(\frac{\pi}{2}, \pi\right)$

16. Find the area of triangle with vertices (1 ,1) , (2, 2) and (3, 3). [1]

a) 1

b) 3

c) 0

d) 2

17. A solution of the differential equation $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$ is [1]

a) $y = 2x^2 - 4$

b) y = 2x

c) y = 2

d) y = 2x - 4

18. **Assertion (A):** If two positive numbers are such that sum is 16 and sum of their cubes is minimum, then numbers are 8, 8. [1]
Reason (R): If f be a function defined on an interval I and c ∈ l and let f be twice differentiable at c, then x = c is a point of local minima if f'(c) = 0 and f''(c) > 0 and f(c) is local minimum value of f.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

19. Range of $\sin^{-1}x$ is [1]

a) None of these

b) $[0, \pi]$

c) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

d) $\left[0, \frac{\pi}{2}\right]$

20. **Assertion (A):** The matrix A = $\begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$ is singular. [1]

Reason (R): A square matrix A is said to be singular, if |A| - 0.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. For the principal values, evaluate $\sin^{-1}\left[\cos\left\{2cosec^{-1}(-2)\right\}\right]$ [2]

22. Solve $\frac{dy}{dx}$ = y cot 2x, given that y = 2 when x = $\frac{\pi}{4}$ [2]

23. If A = $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, find adj A. [2]

OR

Find the adjoint of the given matrix and verify in case that A · (adj A) = (adj A) · A= |A| · I

$\begin{bmatrix} 2 & 3 \\ 5 & 9 \end{bmatrix}$

24. A can hit a target 4 times in 5 shots, B 3 times in 4 shots, and C 2 times in 3 shots. Calculate the probability that any two of A, B, and C will hit the target. [2]

25. P and Q are two points with position vectors $(3\vec{a} - 2\vec{b})$ and $(\vec{a} + \vec{b})$ respectively. Write the position vector of a point R which divides the line segment PQ in the ratio 2: 1 externally. [2]

Section C

26. Evaluate $\int_0^{\pi} \frac{x}{1+\sin x} dx$. [3]

27. Solve the differential equation $x \log|x| \frac{dy}{dx} + y = \frac{2}{x} \log|x|$. [3]

OR

Solve the differential equation: 2xy$\frac{dy}{dx}$ = x^2 + y^2

28. Evaluate: $\int \frac{(5x^2 - 18x + 17)}{(x-1)^2(2x-3)} dx$. [3]

OR

Evaluate: $\int \frac{x^2+1}{(x^2+4)(x^2+25)} dx$

29. If $\vec{a} = (4\hat{i} + 5\hat{j} - \hat{k}), \vec{b} = (\hat{i} - 4\hat{j} + 5\hat{k})$, and $\vec{c} = (3\hat{i} + \hat{j} - \hat{k})$, find a vector $\vec{d}$ which is perpendicular to both a and b and for which $\vec{c} \cdot \vec{d} = = 21$. [3]

OR

if $|\vec{a} + \vec{b}| = 60, |\vec{a} - \vec{b}| = 40$ and $|\vec{b}| = 46,$ find $|\vec{a}|$

30. Differentiate the function $x^{x^2-3} + (x-3)^{x^2}$, for $x > 3$ w.r.t to x. [3]

31. Find the area of the region bounded by y = $|x - 1|$ and y = 1. [3]

Section D

32. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a - b| is even}, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of {2, 4}. [5]

OR

Give an example of a map

i. which is one-one but not onto

ii. which is not one-one but onto

iii. which is neither one-one nor onto.

33. Find the image of the point with position vector $3\hat{i} + \hat{j} + 2\hat{k}$ in the plan $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 4$ Also, find the position vectors of the foot of the perpendicular and the equation of the perpendicular line through $3\hat{i} + \hat{j} + 2\hat{k}$. [5]

OR

Find the vector equation of the line passing through (1,2,3) and || to the plane $\vec{r}.\left(\hat{i} - \hat{j} + 2\hat{k}\right) = 5$ and $\vec{r}.\left(3\hat{i} + \hat{j} + \hat{k}\right) = 6$

34. Minimize z = 3x + 5y subject to the constraints x + 2y $\leq$ 2000, x + y $\leq$ 1500, y $\leq$ 600, x $\geq$ 0 and y $\geq$0 [5]

35. Differentiate the function with respect to x: $\left(x + \frac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$ [5]

Section E

36. **Read the text carefully and answer the questions:** [4]

In a street two lamp posts are 600 feet apart. The light intensity at a distance d from the first (stronger) lamp post is $\frac{1000}{d^2}$, the light intensity at distance d from the second (weaker) lamp post is $\frac{125}{d^2}$ (in both cases the light intensity is inversely proportional to the square of the distance to the light source). The combined light intensity is the sum of the two light intensities coming from both lamp posts.

(i) If l(x) denotes the combined light intensity, then find the value of x so that I(x) is minimum.

(ii) Find the darkest spot between the two lights.

(iii) If you are in between the lamp posts, at distance x feet from the stronger light, then write the combined light intensity coming from both lamp posts as function of x.

OR

Find the minimum combined light intensity?

37. **Read the text carefully and answer the questions:** **[4]**

Three car dealers, say A, B and C, deals in three types of cars, namely Hatchback cars, Sedan cars, SUV cars. The sales figure of 2019 and 2020 showed that dealer A sold 120 Hatchback, 50 Sedan, 10 SUV cars in 2019 and 300 Hatchback, 150 Sedan, 20 SUV cars in 2020; dealer B sold 100 Hatchback, 30 Sedan, 5 SUV cars in 2019 and 200 Hatchback, 50 Sedan, 6 SUV cars in 2020; dealer C sold 90 Hatchback, 40 Sedan, 2 SUV cars in 2019 and 100 Hatchback, 60 Sedan, 5 SUV cars in 2020.

(i) Write the matrix summarizing sales data of 2019 and 2020.

(ii) Find the matrix summarizing sales data of 2020.

(iii) Find the total number of cars sold in two given years, by each dealer?

OR

If each dealer receives a profit of ₹ 50000 on sale of a Hatchback, ₹100000 on sale of a Sedan and ₹200000 on sale of an SUV, then find the amount of profit received in the year 2020 by each dealer.

38. **Read the text carefully and answer the questions:** **[4]**

Shama is studying in class XII. She wants do graduate in chemical engineering. Her main subjects are mathematics, physics, and chemistry. In the examination, her probabilities of getting grade A in these subjects are 0.2, 0.3, and 0.5

respectively.

(i) Find the probability that she gets grade A in all subjects.

(ii) Find the probability that she gets grade A in no subjects.

SOLUTION

Section A

1. **(d)** $2\sqrt{6}$

   Explanation: $2\sqrt{6}$

2. **(a)** $2x \tan^{-1}x - \log(1 + x^2) + C$

   Explanation: Put $x = \tan t$ and $dx = \sec^2 t\, dt$

   Then, $I = \int \cos^{-1}\left(\frac{1-\tan^2 t}{1+\tan^2 t}\right) \sec^2 t\, dt = \int \cos^{-1}(\cos 2t) \sec^2 t\, dt$

   $= 2\int tI\ sec^2II\ t\ dt$

   after solving we get $I = -2x \tan^{-1}x - \log(1 + x^2) + C$

3. **(a)** an ellipse

   Explanation: Given vectors $\hat{i} - 2x\hat{j} + 3y\hat{k}$ and $\hat{i} + 2x\hat{j} - 3\hat{k}$ are perpendicular to each other, so their dot product is zero

   $\Rightarrow 1-4x^2-9y^2 = 0 \Rightarrow 4x^2+9y^2=1$

   $\therefore$ it is an ellipse

4. **(a)** 0.0875

   Explanation: Here, there are total 5 ways by which India can get at least 7 points.
   2 points + 2 points + 2 points + 2 points $=(0.5 \times 0.5 \times 0.5 \times 0.5)$
   1 point + 2 points + 2 points + 2 points $= (0.05 \times 0.5 \times 0.5 \times 0.5)$
   2 points + 1 point + 2 points + 2 points $= (0.5 \times 0.05 \times 0.5 \times 0.5)$
   2 points + 2 points + 1 point + 2 points $=(0.5 \times 0.5 \times 0.05 \times 0.5)$
   P(atleast 7 points) $= 0.5 \times 0.5 \times 0.5 \times 0.5 + 4[0.05 \times 0.5 \times 0.5 \times 0.5]$
   $= 0.0625 + 4(0.00625)$
   $= 0.0625 + 0.025$
   $= 0.0875$

5. **(b)** $\frac{3}{5}$

   Explanation: $P\left(B/A'\right) = \frac{P\left(B \cap A'\right)}{P(A')} = \frac{P(B) - P(B \cap A)}{1 - P(A)}$

   $= \frac{\frac{3}{5} - \frac{3}{10}}{1 - \frac{1}{2}} = \frac{\frac{6-3}{10}}{\frac{1}{2}} = \frac{6}{10} = \frac{3}{5}$

6. **(b)** $\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right)+C$

Explanation: Consider $\int\frac{dx}{4x^2-4x+3}$,

Completing the square

$$4x^2 - 4x + 3 = 4\left(x^2-x+\frac{3}{4}\right)$$

$$= 4\left(x^2-x+\frac{3}{4}+\frac{1}{4}-\frac{1}{4}\right)$$

$$= 4\left(\left(x-\frac{1}{2}\right)^2+\frac{1}{2}\right)$$

$$= \frac{1}{4}\int\frac{dx}{\left(\left(x-\frac{1}{2}\right)^2+\frac{1}{2}\right)}$$

Let $x-\frac{1}{2}=t$

dx = dt

$$\therefore I = \frac{1}{4}\int\frac{dt}{t^2+\frac{1}{\sqrt{2}}^2}$$

We know, $\int\frac{1}{x^2+a^2}=\frac{1}{a}\tan^{-1}\frac{x}{a}+c$

$$\Rightarrow I = \frac{\sqrt{2}}{4}\tan^{-1}\frac{t}{\frac{1}{\sqrt{2}}}+c$$

$$= \frac{1}{2\sqrt{2}}\tan^{-1}\sqrt{2}t+c$$

put t = x - 1

$$= \frac{1}{2\sqrt{2}}\tan^{-1}\frac{2x-1}{\sqrt{2}}+c$$

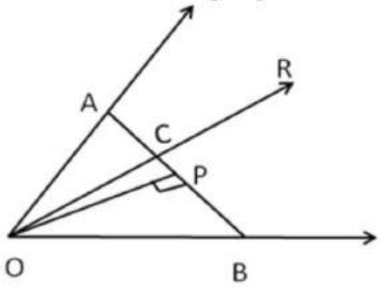
7. **(b)** πab

Explanation: Area of standard ellipse is given by :πab.

8. **(a)** 3 AC = 5 CB

Explanation:

Draw ON, the perpendicular to the lien AB



Let $\vec{i}$ be the unit vector along ON

The resultant force $\vec{R} = 3\overrightarrow{OA} + 5\overrightarrow{OB}$

The angles between $\vec{i}$ and the forces $\vec{R}, 3\overrightarrow{OA}, 5\overrightarrow{OB}$ are ∠CON, ∠AON, ∠BON respectively.

$\Rightarrow R\cos\angle COP = 3OA \times \frac{\text{OP}}{\text{OA}} + 5\text{OB} \times \frac{\text{OP}}{\text{OB}}$

$\frac{R}{OD} = 3 + 5$

$R = 8\overrightarrow{\text{OC}}$

$\overrightarrow{\text{OA}} = \overrightarrow{\text{OC}} + \overrightarrow{\text{CA}}$

$\Rightarrow 3\overrightarrow{\text{OA}} = 3\overrightarrow{\text{OC}} + 3\overrightarrow{\text{CA}}$ ...(i)

$\overrightarrow{\text{OB}} = \overrightarrow{\text{OC}} + \overrightarrow{\text{CB}}$

$\Rightarrow 5\overrightarrow{\text{OB}} = 5\overrightarrow{\text{OC}} + 5\overrightarrow{\text{CB}}$ ...(ii)

$\vec{R} = 8\overrightarrow{OC} + 3\overrightarrow{CA} + 5\overrightarrow{CB}$

$8\overrightarrow{\text{OC}} = 3\overrightarrow{\text{CA}} + 5\overrightarrow{\text{CB}}$

$|3\overrightarrow{CA}| = |5\overrightarrow{CB}|$

$\Rightarrow$ 3CA = 5CB

9. **(a)** 3, 1, - 2

Explanation: We have,

x - y + z - 5 = 0 = x - 3y - 6

$\Rightarrow$ x - 3y - 6 = 0

&, x - y + z - 5 = 0

$\Rightarrow$ x = 3y + 6 ....(i)

x - y + z - 5 = 0 ...(ii)

From (i) and (ii)

We get, 3y + 6 - y + z - 5 = 0

$\Rightarrow$ 2y + z + 1 = 0

$\Rightarrow$ $y = \frac{-z-1}{2}$

$y = \frac{x-6}{3}$ [from (i)]

$\therefore \frac{x-6}{3} = y = \frac{-z-1}{2}$

So, the given equation can be re-written as

$\frac{x-6}{3} = \frac{y}{1} = \frac{z+1}{-2}$

Hence, the direction ratios of the given line are proportional to 3, 1, -2.

10. **(b)** parabolas

Explanation: Given equation can be written as

$\frac{2dy}{y+3} = \frac{dx}{x}$

$\Rightarrow$ 2log (y + 3) = log x + log c

$\Rightarrow$ $(y + 3)^2$ = cx which represents the family of parabolas

11. **(c)** 20π sq. units

Explanation: The area of the standard ellipse is given by ; πab. Here, a = 5 and b = 4 Therefore, the area of curve is $\pi(5)(4) = 20\pi$.

12. **(d)** $\frac{1}{2}(1 + x^2)\tan^{-1} x - \frac{1}{2} x + C$

Explanation: $I = \int xII\left(\tan I^{-1} x\right)dx$

By using product rule we get,

$I = \frac{1}{2}(1 + x^2)\tan^{-1} x - \frac{1}{2} x + C$

13. **(b)** A^t

Explanation: To find:

$$\text{Thus, } A + 2A' = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 3 \\ -2 & -3 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 2 & -4 \\ -2 & 0 & -6 \\ 4 & 6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -3 \\ 2 & 3 & 0 \end{bmatrix} = A'$$

14. **(a)** K^{n-1} Adj. A

Explanation: Adj. (KA) = K^{n-1} Adj.A, where K is a scalar and A is a n $\times$ n matrix.

15. **(a)** $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Explanation: $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$Given function,f(x) is sin x

$\therefore$ f'(x) = cos x
$\Rightarrow$ f'(x) = cos x
= 0

$\Rightarrow$ for $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

f'(x) is increasing,we

$\therefore$ f(x) is increasing in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

Which is the required solution.

16. **(c)** 0

Explanation: AREA OF TRIANGLE=

$\frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$ (Since C_1 and C_2 are identical)

So, value of determinant = 0
Hence, area of triangle = 0

17. **(d)** y = 2x - 4

Explanation: Let, $\frac{dy}{dx} = p$

$\therefore p^2 - xp + y = 0$

$y = xp - p^2$.... (i)

$\Rightarrow \frac{dy}{dx} = (x - 2p)\frac{dy}{dx} + p$

$\Rightarrow p = (x - 2p)\frac{dp}{dx} + p$

$\therefore \frac{dp}{dx} = 0$

$\Rightarrow$ P is constant

from Eqn. (i), $y = x \cdot c - c^2$

$\therefore$ $y = 2x - 4$ is the correct option

18. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Let one number be x, then the other number will be (16 - x).
Let the sum of the cubes of these numbers be denoted by S.

Then, $S = x^3 + (16 - x)^3$

On differentiating w.r.t. x, we get

$\frac{dS}{dx} = 3x^2 + 3(16 - x)^2(-1)$

$= 3x^2 - 3(16 - x)^2$

$\Rightarrow \frac{d^2S}{dx^2} = 6x + 6(16 - x) = 96$

For minima put $\frac{dS}{dx} = 0$.

$\therefore\ 3x^2 - 3(16 - x)^2 = 0$

$\Rightarrow\ x^2 - (256 + x^2 - 32x) = 0$

$\Rightarrow\ 32x = 256$

$\Rightarrow\ x = 8$

At $x = 8, \left(\frac{d^2S}{dx^2}\right)_{x=8} = 96 > 0$

By second derivative test, x = 8 is the point of local minima of S.

Thus, the sum of the cubes of the numbers is the minimum when the numbers are 8 and 16 - 8 = 8

Hence, the required numbers are 8 and 8.

19. **(c)** $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

Explanation:

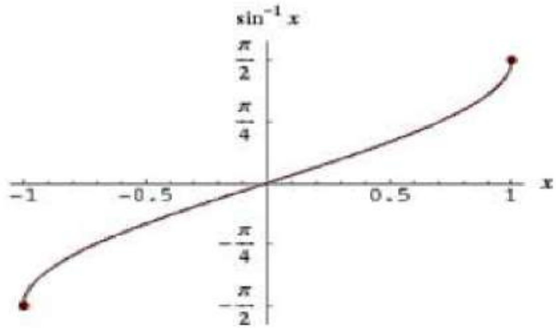
To Find: The range of $\sin^{-1}x$

Here, the inverse function is given by $y = f^{-1}(x)$

The graph of the function $y = \sin^{-1}(x)$ can be obtained from the graph of $Y = \sin x$ by interchanging x and y axes.i.e, if (a, b) is a point on $Y = \sin x$ then (b, a) is

The point on the function $y = \sin^{-1}(x)$

Below is the Graph of range of $\sin^{-1}(x)$



From the graph, it is clear that the range $\sin^{-1}(x)$ is restricted to the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: The determinant of the matrix $A = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$ is $|A| = \begin{vmatrix} 1 & 2 \\ 4 & 8 \end{vmatrix} = 8 - 8 = 0$

Hence, A is a singular matrix.

Section B

21. First of all we need to find the principal value for $\operatorname{cosec}^{-1}(-2)$

Let,

$\operatorname{cosec}^{-1}(-2) = y$

$\Rightarrow \operatorname{cosec} y = -2$

$\Rightarrow -\operatorname{cosec} y = 2$

$\Rightarrow -\operatorname{cosec} \frac{\pi}{6} = 2$

As we know that $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

$\therefore -\operatorname{cosec} \frac{\pi}{6} = \operatorname{cosec}\left(\frac{-\pi}{6}\right)$

The range of principal value of $\operatorname{cosec}^{-1}$ (-2) is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ and

$\operatorname{cosec}\left(\frac{-\pi}{6}\right) = -2$

Thus,the princi value of $\operatorname{cosec}^{-1}(-2)$ is $\frac{-\pi}{6}$.

$\therefore$ Now, the question changes to

$\operatorname{Sin}^{-1}[\cos\frac{-\pi}{6}]$

$\operatorname{Cos}(-\theta) = \cos(\theta)$

$\therefore$ we can write the above expression as

$\operatorname{Sin}^{-1}[\cos\frac{\pi}{6}]$

Let,

$\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right) = Y$

$\Rightarrow \sin y = \frac{\sqrt{3}}{2}$

$\Rightarrow Y = \frac{\pi}{3}$

The range of principal value of $\sin^{-1}$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ and $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

Therefore, the principal value of $\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $\frac{\pi}{3}$

Hence, the principal value of the given equation is $\frac{\pi}{3}$

22. Here,it is given that $\frac{dy}{dx} = y\cot 2x$

Given that, y = 2 when x = $\frac{\pi}{2}$

$\Rightarrow \frac{dy}{y} = y\cot 2x$

$\Rightarrow \frac{dy}{y} = \cot 2x dx$

$\Rightarrow \int \frac{dy}{y} = \int \cot 2x\, dx$

$\Rightarrow \log y = -\frac{\log\ (\sin 2x)}{2} + c$

$\Rightarrow$ log2 + 0 + c

$\Rightarrow$ Thus, C = log2

The particular solution is :- $\log \frac{y}{\sqrt{\sin 2x}} = \log 2$

$\therefore y = 2\sqrt{\sin 2x}$

23. Clearly, $|A| = \begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix}$

The cofactors of the elements of IA I are given by

$A_{11} = 3, A_{12} = -1;$

$A_{21} = -5, A_{22} = 2$

Therefore we have.

$$\text{adj } A = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}' = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

OR

We know that, $\text{adj } A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$

Cofactor of $C_{11} = (-1)^{1-1}(9) = 9$; $C_{12} = (-1)^{1-2}(5) = -5$

$C_{21} = (-1)^{2+1}(3) = -3$; $C_{22} = (-1)^{2+2}(2) = 2$

Therefore we have, $\text{adj } A = \begin{bmatrix} 9 & -5 \\ -3 & 2 \end{bmatrix}^T = \begin{bmatrix} 9 & -3 \\ -5 & 2 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & 3 \\ 5 & 9 \end{vmatrix} \Rightarrow |A| = [2(9) - 5(3)] \Rightarrow |A| = (18 - 15) \Rightarrow |A| = 3$

$\therefore (\text{adj } A) \cdot A = \begin{bmatrix} 9 & -3 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 5 & 9 \end{bmatrix} = \begin{bmatrix} 18-15 & 27-27 \\ -10+10 & -15+18 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

and $A(\text{adj } A) = \begin{bmatrix} 2 & 3 \\ 5 & 9 \end{bmatrix}\begin{bmatrix} 9 & -3 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 18-15 & -6+6 \\ 45-45 & -15+18 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

Also, $|A| \cdot I = 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

Hence, $A \cdot (\text{Adj } A) = (\text{Adj } A) \cdot A = |A| \cdot I$

24. Consider the following events:

E = A hits the target, F = B hits the target, and G = C hits the target

We have, $P(E) = \frac{4}{5}, P(F) = \frac{3}{4}$ and $P(G) = \frac{2}{3}$

Required probability = P (Any two of A, B and C will hit the target)

$= P(E \cap F \cap \bar{G}) \cup (\bar{E} \cap F \cap G) \cup (E \cap \bar{F} \cap G)]$

$= P(E \cap F \cap \bar{G}) + P(\bar{E} \cap F \cap G) + P(E \cap \bar{F} \cap G)$ [independent events]

$= P(E)P(F)P(\bar{G}) + P(\bar{E})P(F)P(G) + P(E)P(\bar{F})P(G)$ [independent events]

$= \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3} + \frac{1}{5} \times \frac{3}{4} \times \frac{2}{3} + \frac{4}{5} \times \frac{1}{4} \times \frac{2}{3} = \frac{13}{30}.$

25. Here,The position vectors of the given points are

$P(3\vec{a} - 2\vec{b})$ and $Q(\vec{a} + \vec{b})$.

We have to divide PQ in the ratio 2: 1 externally at the point R

The position vector of R is

$$\frac{2(\vec{a}+\vec{b})-1\cdot(3\vec{a}-2\vec{b})}{(2-1)} = (-\vec{a}+4\vec{b}).$$

Therefore, the position vector of R is $(-\vec{a}+4\vec{b})$.

Section C

26. Given $I = \int_0^{\pi}\frac{x}{1+\sin x}dx$ ....(i)

$\therefore \quad I = \int_0^{\pi}\frac{\pi-x}{1+\sin(\pi-x)}dx\left[\because \int_0^{a}f(x)dx = \int_0^{a}f(a-x)dx\right]$

$\Rightarrow \quad I = \int_0^{\pi}\frac{\pi-x}{1+\sin x}dx$ .....(ii)[$\because \sin(\pi-x) = \sin x$]

Adding Equations (i) and (ii) we get ,

$2I = \int_0^{\pi}\frac{x+\pi-x}{1+\sin x}dx$

$= \pi\int_0^{\pi}\frac{1}{1+\sin x}dx$

$= \pi\int_0^{\pi}\frac{1}{1+\sin x}\times\frac{1-\sin x}{1-\sin x}dx$

$= \pi\int_0^{\pi}\frac{1-\sin x}{1-\sin^2 x}dx$

$= \pi\int_0^{\pi}\frac{1-\sin x}{\cos^2 x}dx$ $[cos^2x + sin^2x = 1 \Rightarrow cos^2x = 1 - sin^2x]$

$= \pi\int_0^{\pi}\left(\sec^2 x - \tan x\sec x\right)dx$

$= \pi[\tan x - \sec x]_0^{\pi}$

$= \pi[(\tan\pi - \sec\pi) - (\tan 0 - \sec 0)]$

$= \pi$ [(0 - (-1) - (0 - 1)]

$= \pi(1+1)$

$\Rightarrow 2I = 2\pi \quad \Rightarrow I = \pi$

27. We have to solve,

$x\log|x|\frac{dy}{dx} + y = \frac{2}{x}\log|x|$

On dividing both sides by x log x, we get

$\frac{dy}{dx} + \frac{y}{x\log|x|} = \frac{2}{x^2}\frac{\log|x|}{\log|x|} = \frac{2}{x^2}$

which is a linear differential equation of first order, which is of the form of

$\frac{dy}{dx} + Py = Q,$

Here, $P = \frac{1}{x\log|x|}$ and $Q = \frac{2}{x^2}$

We know that ,

$$\text{IF} = e^{\int P dx} = e^{\int \frac{1}{x\log |x|} dx}$$

put $\log|x| = t \Rightarrow \frac{1}{x}dx = dt$

$$\therefore IF = \int \frac{1}{t}dt = \log|t| = \log|\log x|$$

$$\Rightarrow \quad \text{IF} = \log|x| \quad \left[\because e^{\log x} = x\right]$$

Now, solution of above equation is given by

$y \times \text{IF} = \int (Q \times \text{IF}) dx + C$

$$y\log|x| = \int \underset{II}{\frac{2}{x^2}} \underset{I}{\log|x|} dx + C$$

$$\Rightarrow \quad y\log|x| = 2\left[\log|x| \int \frac{1}{x^2}dx - \int \left(\frac{d}{dx}(\log|x|) \cdot \int \frac{1}{x^2}dx\right)dx\right] + C$$ [using integration by parts]

$$\Rightarrow \quad y\log|x| = 2\left[\log|x| \cdot \left(-\frac{1}{x}\right) - \int \frac{1}{x} \cdot \left(-\frac{1}{x}\right)dx + C\right.$$

$$\Rightarrow \quad y\log|x| = 2\left[-\frac{1}{x}\log|x| + \int \frac{1}{x^2}dx\right] + C$$

$$\therefore \quad y\log|x| = -\frac{2}{x}\log|x| - \frac{2}{x} + C$$

OR

The given differential equation is,

$$2xy\frac{dy}{dx} = x^2 + y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$$

This is a homogeneous differential equation

Putting $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$, we get

$$v + x\frac{dv}{dx} = \frac{x^2 + v^2x^2}{2x^2v}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1 + v^2}{2v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+v^2}{2v} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1-v^2}{2v}$$

$$\Rightarrow \frac{2v}{1-v^2}dv = \frac{1}{x}dx$$

Integrating both sides, we get

$$\int \frac{2v}{1-v^2}dv = \int \frac{1}{x}dx$$

$\Rightarrow$ - log |1 - v^2| = log |x| + log C

$\Rightarrow$ - log |(1 - v^2)|x| = log C

Putting v = $\frac{y}{x}$, we get

$$\Rightarrow -\log \left| \left(\frac{x^2-y^2}{x^2} \right) x \right| = \log C$$

$$\Rightarrow \left| \frac{x}{x^2-y^2} \right| = C$$

$\Rightarrow$ |x| = C| (x^2 - y^2)|

28. Let the given integral be, $I = \int \frac{5x^2-18x+17}{(x-1)^2(2x-3)}dx$

Now using partial fractions putting, $\frac{5x^2-18x+17}{(x-1)^2(2x-3)}$

$$= \frac{A}{(2x-3)} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

A(x - 1)^2 + B(2x - 3)(x - 1) + C(2x - 3) = 5x^2 - 18x + 17

Putting x - 1 = 0,

X = 1

A(0) + B(0) + C(2 - 3) = 5 - 18 + 17

C(-1) = 4

Putting 2x - 3 = 0,

$$x = \frac{3}{2}$$

$$A\left(\frac{3}{2} - 1\right)^2 + B(0) + C(0) = 5\left(\frac{3}{2}\right)^2 - 18\left(\frac{3}{2}\right) + 17$$

$$A\left(\frac{1}{4}\right)+0=5\times\frac{9}{4}-27+17$$

$$A\left(\frac{1}{4}\right)=\frac{45}{4}-10=\frac{5}{4}$$

A = 5

By equating the coefficient of x^2, we get ,

A + 2B = 5

5 + 2B = 5

2B = 0

B = 0

From equation (1), we get,

$$\frac{5x^2-18x+17}{(x-1)^2(2x-3)}=5\times\frac{1}{(2x-3)}+0-4\times\frac{1}{(x-1)^2}$$

$$\int\frac{5x^2-18x+17}{(x-1)^2(2x-3)}dx=\frac{5}{2}\log(2x-3)+\frac{4}{x-1}+c$$

OR

Consider the integral

$$I=\int\frac{x^2+1}{\left(x^2+4\right)\left(x^2+25\right)}dx$$

Let $y = x^2$

Thus,

$$\frac{x^2+1}{\left(x^2+4\right)\left(x^2+25\right)}=\frac{y+1}{(y+4)(y+25)}$$

$$\Rightarrow\ \frac{y+1}{(y+4)(y+25)}=\frac{A}{y+4}+\frac{B}{y+25}$$

by using partial fraction

$$\Rightarrow\ \frac{y+1}{(y+4)(y+25)}=\frac{A(y+25)+B(y+4)}{(y+4)(y+25)}$$

$\Rightarrow$ y + 1 = Ay + 25A + By + 4B

Comparing the coefficients,we have,

A + B = 1 and 25A + 4B = 1

Solving the above equation, We have,

$$A=\frac{-1}{7}\text{ and }B=\frac{8}{7}$$

Thus using values of A, and B, we get, $I = \int \frac{x^2+1}{\left(x^2+4\right)\left(x^2+25\right)} dx$

$= \int \frac{\frac{-1}{7}}{x^2+4} dx + \int \frac{\frac{8}{7}}{x^2+25} dx$

$= \frac{-1}{7} \int \frac{1}{x^2+4} dx + \frac{8}{7} \int \frac{1}{x^2+25} dx$

$= \frac{-1}{7} \times \frac{1}{2} \tan^{-1} \frac{x}{2} + \frac{8}{7} \times \frac{1}{5} \tan^{-1} \frac{x}{5} + C$

$= \frac{-1}{14} \tan^{-1} \frac{x}{2} + \frac{8}{35} \tan^{-1} \frac{x}{5} + c$

29. Given vectors are ;

$\vec{a} = (4\hat{i} + 5\hat{j} - \hat{k}), \vec{b} = (\hat{i} - 4\hat{j} + 5\hat{k})$ and $\vec{c} = 3\hat{i} + \hat{j} - \hat{k}$

$\because \vec{d} \perp \vec{a}$ and $\vec{b}$

$\Rightarrow \vec{d} \| (\vec{a} \times \vec{b}) \Rightarrow \vec{d} = k(\vec{a} \times \vec{b})$ (where k is any non zero real number)

$\Rightarrow \vec{d} = k \{(25 - 4) \vec{i} - (20 + 1) \vec{j} + (-16 - 15) \vec{k}\} \Rightarrow \vec{d} = 21 k \vec{i} - 21 k \vec{j} - 21 k\vec{k}\}$

Now, $\vec{c} \cdot \vec{d} = 21$

$\Rightarrow (3i + j - k) \cdot (21ki - 21kj - 21kk) = 21 \Rightarrow 63 k - 21 k = 21 \Rightarrow k = \frac{1}{3}$

$\therefore \vec{d} = 21 \times \frac{1}{3}\hat{i} - 21 \times \frac{1}{3}\hat{j} - 21 \times \frac{1}{3}\hat{k} = \vec{d} = (7\hat{i} - 7\hat{j} - 7\hat{k})$

OR

Given,

$|\vec{a} + \vec{b}| = 60$

squaring both the sides,

$|\vec{a} + \vec{b}|^2 = (60)^2$

$\Rightarrow (\vec{a} + \vec{b})^2 = (60)^2$

$\Rightarrow (\vec{a})^2 + (\vec{b})^2 + 2\vec{a}.\vec{b} = 3600$ ... (i)

Also given,

$|\vec{a} - \vec{b}| = 40$

squaring both the sides

$|\vec{a} - \vec{b}|^2 = (40)^2$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b} = 1600$ .... (ii)

Adding equations (i) and (ii), we get :-

$2|\vec{a}|^2 + 2|\vec{b}|^2 + 2\vec{a}.\vec{b} - 2\vec{a}.\vec{b} = 3600 + 1600$

$2|\vec{a}|^2 + 2(46)^2 = 5200$ [given, |b|=46]

$2|\vec{a}|^2 = 5200 - 4232$

$2|\vec{a}|^2 = 968$

$|\vec{a}|^2 = \frac{968}{2}$

$|\vec{a}|^2 = 484$

$\vec{a} = \sqrt{484}$

$|a| = 22$

30. Let $y = x^{x^2-3} + (x-3)^{x^2}$

And let $u = x^{x^2-3}$ and $v = (x-3)^{x^2}$

$\therefore\ y = u + v$

Differentiating both sides w.r.t. x we get

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$.....(i)

Now,

$u = x^{x^2-3}$

Taking logarithm both sides

$\log u = \log x^{x^2-3}$

$\Rightarrow \log u = (x^2 - 3) \log x$

Differentiating w.r.t. x, we get

$\frac{1}{u}\frac{du}{dx} = \log x \times \frac{d}{dx}\left(x^2-3\right) + \left(x^2-3\right) \times \frac{d}{dx}(\log x)$

$\Rightarrow \frac{du}{dx} = u\left[\log x \times 2x + \left(x^2-3\right) \times \frac{1}{x}\right]$

$\Rightarrow \frac{du}{dx} = x^{x^2-3}\left[\frac{x^2-3}{x} + 2x\log x\right]$.....(ii)

Also,

$v = (x-3)^{x^2}$

Taking logarithm both sides

$\log v = \log (x-3)^{x^2}$

$\Rightarrow \log v = x^2 \log(x-3)$

Differentiating both sides w.r.t. x

$\frac{1}{v}\frac{dv}{dx} = \log(x-3) \times \frac{d}{dx}\left(x^2\right) + x^2 \times \frac{d}{dx}[\log(x-3)]$

$$\Rightarrow \frac{dv}{dx} = v\left[\log(x-3) \times 2x + x^2 \times \frac{1}{(x-3)} \times \frac{d}{dx}(x-3)\right]$$

$$\Rightarrow \frac{dv}{dx} = (x-3)^{x^2}\left[2x\log(x-3) + \frac{x^2}{(x-3)} \times 1\right]$$

$$\Rightarrow \frac{dv}{dx} = (x-3)^{x^2}\left[\frac{x^2}{(x-3)} + 2x\log(x-3)\right] \text{....(iii)}$$

Substituting (ii) and (iii) in (i)

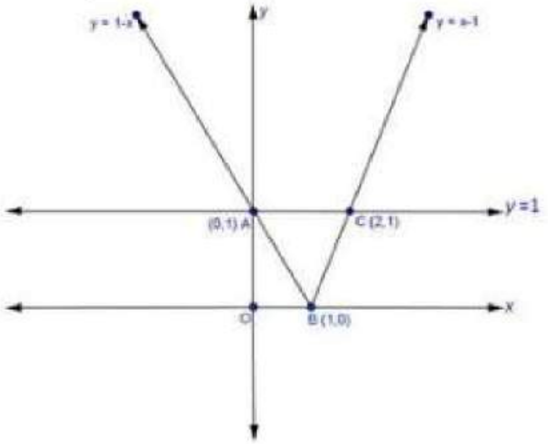
$$\therefore \frac{dy}{dx} = x^{x^2-3}\left[\frac{x^2-3}{x} + 2x\log x\right] + (x-3)^{x^2}\left[\frac{x^2}{(x-3)} + 2x\log(x-3)\right]$$

31. To find area bounded by y = 1 and

$y = |x-1|$

$$y = \begin{cases} x-1, & \text{if } x \geq 0 \quad \ldots(1) \\ 1-x, & \text{if } x < 0 \quad \ldots(2) \end{cases}$$

A rough sketch of the curve is as under:-



Bounded reigon is the required region. So

Required area of bounded region=Area of Region ABCA

A = Region ABDA + Region BCDB

$$= \int_0^1 (y_1 - y_2)dx + \int_1^2 (y_1 - y_3)dx$$

$$= \int_0^1 (1 - 1 + x)dx + \int_1^2 (1 - x + 1)dx$$

$$= \int_0^1 x dx + \int_1^2 (2 - x)dx$$

$$= \left(\frac{x^2}{2}\right)_0^1 + \left(2x - \frac{x^2}{2}\right)_1^2$$

$$= \left(\frac{1}{2} - 0\right) + \left[(4-2) - \left(2 - \frac{1}{2}\right)\right]$$

$$= \frac{1}{2} + \left(2 - 2 + \frac{1}{2}\right)$$

A = 1 sq. unit

Section D

32. A = {1, 2, 3, 4, 5} and R = {(a, b) : |a - b| is even}, then R = {(1, 3), (1, 5), (3, 5), (2, 4)}

1. For (a, a), |a - a| = 0 which is even. ∴ R is reflexive.
   If |a - b| is even, then |b - a| is also even. ∴ R is symmetric.
   Now, if |a - b| and |b - c| is even then |a - b + b - c| is even
   ⇒ |a - c| is also even. ∴ R is transitive.
   Therefore, R is an equivalence relation.
2. Elements of {1, 3, 5} are related to each other.
   Since |1 - 3| = 2, |3 - 5| = 2, |1 - 5| = 4 all are even numbers
   ⇒ Elements of {1, 3, 5} are related to each other.
   Similarly elements of (2, 4) are related to each other.
   Since |2 - 4| = 2 an even number, then no element of the set {1, 3, 5} is related to any element of (2, 4).
   Hence no element of {1, 3, 5} is related to any element of {2, 4}.

OR

i. Let $f: N \to N$, be a mapping defined by f (x) = 2x
   Which is one-one
   For $f(x_1) = f(x_2)$
   $\Rightarrow 2x_1 = 2x_2$
   $x_1 = x_2$
   Further **f** is not onto, as for 1 ∈ N, there does not exist any x in N such that f(x) = 1.

ii. Let $f: N \to N$ given by f(1) = f(2) = 1 and f (x) = x - 1 for every x > 2 is onto but not one-one. f is not one-one as **f**(1) = **f**(2) = 1. But **f** is onto.

iii. The mapping $f: R \to R$ defined as **f**(x) = x^2, is neither one-one not onto.

33. Let P be the point with position vector $\vec{p} = 3\hat{i} + \hat{j} + 2\hat{k}$ and M be the image of P in the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 4$.
In addition, let Q be the foot of the perpendicular from P on to the given plane. So, Q is the midpoint of PM.
Direction ratios of PM are proportional to 2, –1, 1 as PM is normal to the plane and parallel to $2\hat{i} - \hat{j} + \hat{k}$.
Recall the vector equation of the line passing through the point with position vector $\vec{r}$ and parallel to vector $\vec{b}$ is given by
$\vec{r} = \vec{a} + \lambda\vec{b}$
Here, $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$

Hence, the equation of PM is

$\vec{r} = (3\hat{i} + \hat{j} + 2\hat{k}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$

$\therefore \vec{r} = (3 + 2\lambda)\hat{i} + (1 - \lambda)\hat{j} + (2 + \lambda)\hat{k}$

Let the position vector of M be $\vec{m}$. As M is a point on this line, for some scalar α, we have

$\Rightarrow \vec{m} = (3 + 2\alpha)\hat{i} + (1 - \alpha)\hat{j} + (2 + \alpha)\hat{k}$

Now, let us find the position vector of Q, the midpoint of PM.

Let this be$\vec{q}$.

Using the midpoint formula, we have

$$\vec{q} = \frac{\vec{p}+\vec{m}}{2}$$

$$\Rightarrow \vec{q} = \frac{[3\hat{i}+\hat{j}+2\hat{k}] + [(3+2\alpha)\hat{i}+(1-\alpha)\hat{j}+(2+\alpha)\hat{k}]}{2}$$

$$\Rightarrow \vec{q} = \frac{(3+(3+2\alpha))\hat{i}+(1+(1-\alpha))\hat{j}+(2+(2+\alpha))\hat{k}}{2}$$

$$\Rightarrow \vec{q} = \frac{(6+2\alpha)\hat{i}+(2-\alpha)\hat{j}+(4+\alpha)\hat{k}}{2}$$

$$\therefore \vec{q} = (3 + \alpha)\hat{i} + \frac{(2-\alpha)}{2}\hat{j} + \frac{(4+\alpha)}{2}\hat{k}$$

This point lies on the given plane, which means this point satisfies the plane equation $\vec{r} \cdot (2\hat{\imath} - \hat{\jmath} + \hat{k}) = 4$.

$$\Rightarrow \left[(3 + \alpha)\hat{i} + \frac{(2-\alpha)}{2}\hat{j} + \frac{(4+\alpha)}{2}\hat{k}\right] \cdot (2\hat{\imath} - \hat{j} + \hat{k}) = 4$$

$$\Rightarrow 2(3 + \alpha) - \left(\frac{2-\alpha}{2}\right)(1) + \left(\frac{4+\alpha}{2}\right)(1) = 4$$

$$\Rightarrow 6 + 2\alpha + \frac{4+\alpha-(2-\alpha)}{2} = 4$$

$\Rightarrow 2\alpha + (1 + \alpha) = -2$

$\Rightarrow 3\alpha = -3$

$\therefore \alpha = -1$

We have the image $\vec{m} = (3 + 2\alpha)\hat{i} + (1 - \alpha)\hat{j} + (2 + \alpha)\hat{k}$

$\Rightarrow \vec{m} = [3 + 2(-1)]\hat{i} + [1 - (-1)]\hat{j} + [2 + (-1)]\hat{k}$

$\therefore \vec{m} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$

Therefore, the image is (1, 2, 1)

The foot of the perpendicular$\vec{q} = (3 + \alpha)\hat{i} + \frac{(2-\alpha)}{2}\hat{j} + \frac{(4+\alpha)}{2}\hat{k}$

$$\Rightarrow \vec{q} = [3 + (-1)]\hat{i} + \left[\frac{2-(-1)}{2}\right]\hat{j} + \left[\frac{4+(-1)}{2}\right]\hat{k}$$

$$\therefore \vec{q} = 2\hat{i} + \frac{3}{2}\hat{j} + \frac{3}{2}\hat{k}$$

Thus, the position vector of the image is$\hat{i} + 2\hat{j} + \hat{k}$ and that of the foot of the perpendicular is$2\hat{i} + \frac{3}{2}\hat{j} + \frac{3}{2}\hat{k}$

OR

Line passing through (1, 2, 3)

ie $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and parallel to the given planes is perpendicular to the vectors

$\vec{b}_1 = \hat{i} - \hat{j} + 2\hat{k}$ and

$\vec{b}_2 = 3\hat{i} + \hat{j} + \hat{k}$

Required line is parallel to $\vec{b}_1 \times \vec{b}_2$

$$\vec{b} = \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 3 & 1 & 1 \end{vmatrix} = \vec{i}(-1-2) - \vec{j}(1-6) + \vec{k}(1+3) = -3\vec{i} + 5\vec{j} + 4\vec{k}$$

Required education of line is : -

$\vec{r} = \vec{a} + \lambda\vec{b}$

$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(-3\hat{i} + 5\hat{j} + 4\hat{k}\right)$

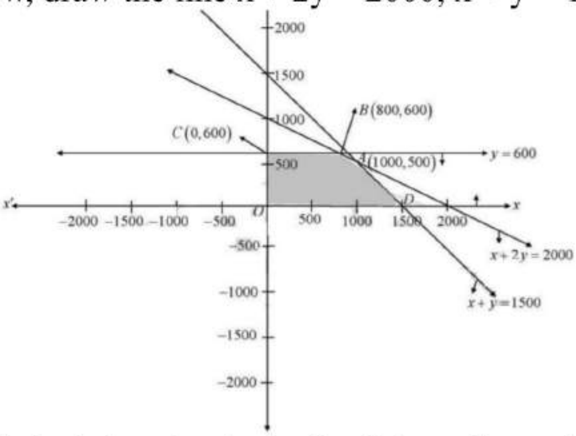
34. Given,

Objective function, Maximize z = 3x + 5y

subject to the constraints

$x + 2y \le 2000, x + y \le 1500, y \le 600, x \ge 0$ and $y \ge 0$

Now, draw the line x + 2y = 2000, x + y = 1500, and y = 600



and shaded region is the feasible region satisfied by above inequalities. Here, the feasible region is bounded.

Now,

Corner points (x,y)	Z =3x+5y

(0,0)	0
(1500,0)	3.1500+5.0=4500
(1000,500)	3.1000+5.500=5500
(0,500)	0+500.5=2500

Hence the maximum value of z is 5500, which occurs at A(1000, 500)

35. Let $y = \left(x + \frac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$

Also, Let $u = \left(x + \frac{1}{x}\right)^x$ and $v = x^{\left(1+\frac{1}{x}\right)}$

$\therefore\ y = u + v$

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ ...(i)

Then, $u = \left(x + \frac{1}{x}\right)^x$

$\Rightarrow \log u = \log\left(x + \frac{1}{x}\right)^x$

$\Rightarrow \log u = x\log\left(x + \frac{1}{x}\right)$

Differentiating both sides with respect to x,

$$\frac{1}{u}\frac{du}{dx} = \log\left(x + \frac{1}{x}\right)\frac{d}{dx}(x) + x\frac{d}{dx}\left[\log\left(x + \frac{1}{x}\right)\right]$$

$$\Rightarrow \frac{1}{u}\frac{du}{dx} = \log\left(x + \frac{1}{x}\right) + x\frac{1}{\left(x+\frac{1}{x}\right)}\frac{d}{dx}\left(x + \frac{1}{x}\right)$$

$$\Rightarrow \frac{du}{dx} = u\left[\log\left(x + \frac{1}{x}\right) + \frac{x}{\left(x+\frac{1}{x}\right)} \times \left(1 - \frac{1}{x^2}\right)\right]$$

$$\Rightarrow \frac{du}{dx} = \left(x + \frac{1}{x}\right)^x \left[\log\left(x + \frac{1}{x}\right) + \frac{\left(x - \frac{1}{x}\right)}{\left(x + \frac{1}{x}\right)}\right]$$

$$\Rightarrow \frac{du}{dx} = \left(x + \frac{1}{x}\right)^x \left[\log\left(x + \frac{1}{x}\right) + \frac{x^2 - 1}{x^2 + 1}\right]$$

$$\Rightarrow \frac{du}{dx} = \left(x + \frac{1}{x}\right)^x \left[\frac{x^2 - 1}{x^2 + 1} + \log\left(x + \frac{1}{x}\right)\right]$$

Again, $v = x\left(1 + \frac{1}{x}\right)$

$$\Rightarrow \log v = \log\left[x\left(1 + \frac{1}{x}\right)\right]$$

$$\Rightarrow \log v = \left(1 + \frac{1}{x}\right)\log x$$

Differentiating both sides with respect to x,

$$\frac{1}{v}\frac{dv}{dx} = \log x \frac{d}{dx}\left(1 + \frac{1}{x}\right) + \left(1 + \frac{1}{x}\right)\frac{d}{dx}\log x$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \left(-\frac{1}{x^2}\right)\log x + \left(1 + \frac{1}{x}\right)\left(\frac{1}{x}\right)$$

$$\Rightarrow \frac{1}{v}\frac{dv}{dx} = -\frac{\log x}{x^2} + \frac{1}{x} + \frac{1}{x^2}$$

$$\Rightarrow \frac{dv}{dx} = v\left[\frac{-\log x + x + 1}{x^2}\right]$$

$$\Rightarrow \frac{dv}{dx} = x\left(1 + \frac{1}{x}\right)\left(\frac{x + 1 - \log x}{x^2}\right) \quad \text{....(iii)}$$

From (i), (ii) and (iii), we obtain

$$\frac{dy}{dx}=\left(x+\frac{1}{x}\right)^x\left[\frac{x^2-1}{x^2+1}+\log\left(x+\frac{1}{x}\right)\right]+x\left(1+\frac{1}{x}\right)\left(\frac{x+1-\log x}{x^2}\right)$$

Section E

36. **Read the text carefully and answer the questions:**

In a street two lamp posts are 600 feet apart. The light intensity at a distance d from the first (stronger) lamp post is $\frac{1000}{d^2}$, the light intensity at distance d from the second (weaker) lamp post is $\frac{125}{d^2}$ (in both cases the light intensity is inversely proportional to the square of the distance to the light source). The combined light intensity is the sum of the two light intensities coming from both lamp posts.

(i) We have, $I(x)=\frac{1000}{x^2}+\frac{125}{(600-x)^2}$

$\Rightarrow I'(x)=\frac{-2000}{x^3}+\frac{250}{(600-x)^3}$ and

$\Rightarrow I''(x)=\frac{6000}{x^4}+\frac{750}{(600-x)^4}$

For maxima/minima, I'(x) - 0

$\Rightarrow \frac{2000}{x^3}=\frac{250}{(600-x)^3} \Rightarrow 8(600-x)^3=x^3$

Taking cube root on both sides, we get

$2(600-x)=x \Rightarrow 1200=3x \Rightarrow x=400$

Thus, I(x) is minimum when you are at 400 feet from the strong intensity lamp post.

(ii) At a distance of 200 feet from the weaker lamp post.

Since I(x) is minimum when x = 400 feet, therefore the darkest spot between the two light is at a distance of 400 feet from a stronger lamp post, i.e., at a distance of 600 - 400 = 200 feet from the weaker lamp post.

(iii) $\frac{1000}{x^2}+\frac{125}{(600-x)^2}$

Since, the distance is x feet from the stronger light, therefore the distance from the weaker light will be 600 - x.

So, the combined light intensity from both lamp posts is given by

$$\frac{1000}{x^2} + \frac{125}{(600-x)^2}.$$

OR

We know that $I(x) = \frac{1000}{x^2} + \frac{125}{(600-x)^2}$

When x = 400

$$I(x) = \frac{1000}{160000} + \frac{125}{(600-400)^2}$$

$$= \frac{1}{160} + \frac{125}{40000} = \frac{1}{160} + \frac{1}{320} = \frac{3}{320} \text{ units}$$

37. **Read the text carefully and answer the questions:**

Three car dealers, say A, B and C, deals in three types of cars, namely Hatchback cars, Sedan cars, SUV cars. The sales figure of 2019 and 2020 showed that dealer A sold 120 Hatchback, 50 Sedan, 10 SUV cars in 2019 and 300 Hatchback, 150 Sedan, 20 SUV cars in 2020; dealer B sold 100 Hatchback, 30 Sedan, 5 SUV cars in 2019 and 200 Hatchback, 50 Sedan, 6 SUV cars in 2020; dealer C sold 90 Hatchback, 40 Sedan, 2 SUV cars in 2019 and 100 Hatchback, 60 Sedan, 5 SUV cars in 2020.



(i) *Hatchback* *Sedan* *SUV*

$$\begin{matrix} A \\ B \\ C \end{matrix} \begin{bmatrix} 120 & 50 & 10 \\ 100 & 30 & 5 \\ 90 & 40 & 2 \end{bmatrix}$$

In 2019, dealer A sold 120 Hatchbacks, 50 Sedans and 10 SUV; dealer B sold 100 Hatchbacks, 30 Sedans and 5 SUVs and dealer C sold 90 Hatchbacks, 40 Sedans and 2 SUVs.

∴ Required matrix, say P, is given by

Hatchback *Sedan* *SUV*

$$P = \begin{matrix} A \\ B \\ C \end{matrix} \begin{bmatrix} 120 & 50 & 10 \\ 100 & 30 & 5 \\ 90 & 40 & 2 \end{bmatrix}$$

In 2020, dealer A sold 300 Hatchbacks, 150 Sedans, 20 SUVs dealer B sold 200 Hatchbacks, 50 sedans, 6 SUVs dealer C sold 100 Hatchbacks, 60 sedans, 5 SUVs.

∴ Required matrix, say Q, is given by

$$Q = \begin{matrix} A \\ B \\ C \end{matrix} \overset{\begin{matrix} Hatchback & Sedan & SUV \end{matrix}}{\begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 60 & 5 \end{bmatrix}}$$

(ii)

$$\begin{matrix} A \\ B \\ C \end{matrix} \overset{\begin{matrix} Hatchback & Sedan & SUV \end{matrix}}{\begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 60 & 5 \end{bmatrix}}$$

In 2020, dealer A sold 300 Hatchback, 150 Sedan, 20 SUV dealer B sold 200 Hatchback, 50 sedan, 6 SUV dealer C sold 100 Hatchback, 60 sedan, 5 SUV.

∴ Required matrix, say Q, is given by

$$Q = \begin{matrix} A \\ B \\ C \end{matrix} \overset{\begin{matrix} Hatchback & Sedan & SUV \end{matrix}}{\begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 60 & 5 \end{bmatrix}}$$

(iii)Total number of cars sold in two given years, by each dealer, is given by

$$P + Q = \begin{matrix} A \\ B \\ C \end{matrix} \overset{\begin{matrix} Hatchback & Sedan & SUV \end{matrix}}{\begin{bmatrix} 120 + 300 & 50 + 150 & 10 + 20 \\ 100 + 200 & 30 + 50 & 5 + 6 \\ 90 + 100 & 40 + 60 & 2 + 5 \end{bmatrix}}$$

$$= \begin{matrix} A \\ B \\ C \end{matrix} \overset{\begin{matrix} Hatchback & Sedan & SUV \end{matrix}}{\begin{bmatrix} 420 & 200 & 30 \\ 300 & 80 & 11 \\ 190 & 100 & 7 \end{bmatrix}}$$

OR

The amount of profit in 2020 received by each dealer is given by the matrix

$$\begin{matrix} A \\ B \\ C \end{matrix} \overset{\begin{matrix} Hatchback & Sedan & SUV \end{matrix}}{\begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 60 & 5 \end{bmatrix}} \begin{bmatrix} 50000 \\ 100000 \\ 200000 \end{bmatrix}$$

$$= \begin{matrix} A \\ B \\ C \end{matrix} \begin{bmatrix} 15000000 + 15000000 + 4000000 \\ 10000000 + 5000000 + 1200000 \\ 5000000 + 6000000 + 1000000 \end{bmatrix}$$

$$= \begin{matrix} A \\ B \\ C \end{matrix} \begin{bmatrix} 34000000 \\ 16200000 \\ 12000000 \end{bmatrix}$$

38. **Read the text carefully and answer the questions:**
Shama is studying in class XII. She wants do graduate in chemical engineering. Her main subjects are mathematics, physics, and chemistry. In the examination, her probabilities of getting grade A in these subjects are 0.2, 0.3, and 0.5 respectively.

(i) P (Grade A in Maths) = P(M) = 0.2
P (Grade A in Physics) = P(P) = 0.3
P (Grade A in Chemistry) = P(C) = 0.5

P(not A garde in Maths) = $P(\overline{M})$ = 1 - 0.2 = 0.8

P(not A garde in Physics) = $P(\overline{P})$ = 1 - 0.3 = 0.7

P(not A garde in Chemistry) = $P(\overline{C})$ = 1 - 0.5 = 0.5
P(getting grade A in all subjects) = P(M ∩ P ∩ C)
= P(M) × P(P) × P(C)
= 0.2 × 0.3 × 0.5 = 0.03

(ii) P (Grade A in Maths) = P(M) = 0.2
P (Grade A in Physics) = P(P) = 0.3
P (Grade A in Chemistry) = P(C) = 0.5

P(not A garde in Maths) = $P(\overline{M})$ = 1 - 0.2 = 0.8

P(not A garde in Physics) = $P(\overline{P})$ = 1 - 0.3 = 0.7

P(not A garde in Chemistry) = $P(\overline{C})$ = 1 - 0.5 = 0.5

P(getting grade A in on subjects) = $P(\overline{M} \cap \overline{P} \cap \overline{C})$

$= P(\overline{M}) \times P(\overline{P}) \times P(\overline{C})$
= 0.8 × 0.7 × 0.5 = 0.280